Power and Sample Size

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Overview

Outline

- Review
- Introduction to power and effect size
- Power and other factors
- Example – estimating sample size

Learning objectives

- to be able to define power
- to be able to name 3 factors that affect power
- to be aware of power and sample size resources

Review

Terminology

- null hypothesis, H_O

  typically states that there is no relationship between the response and explanatory variable(s)

- alternative hypothesis, H_A

  typically states that there is a relationship between the response and explanatory variable(s)

Review

More terminology

- type I error, α
  chance of mistakenly rejecting H_O

- type II error, β
  chance of mistakenly accepting H_O

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Review

Truth table

	State of Reality	
Decision	H_O true	H_O false
Reject H_O	type I error α	correct decision $1-\beta$
Accept H_O	correct decision $1-\alpha$	type II error β

Introduction to power

What is power?

- chance of correctly rejecting H_O

  ie. finding a true significant result

Why is power analysis important?

- helps to design study
- determines if we are able to detect a meaningful effect

Introduction to power

Graphical representation

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Introduction to effect size

What is effect size?

- "the degree to which the null hypothesis is false" (Cohen, 1977)
- refers to the population rather than a specific sample
- effect size is a scale-free, continuous measure
- under H_O, effect size, d, is 0
- each statistical test has its own effect size index

Introduction to effect size

Some tests and their effect sizes

- two sample t-test: difference between means in terms of within group standard deviation
- product-moment correlation: correlation
- one-way analysis of variance: ratio of standard deviation between groups and standard deviation within groups

- see Cohen (1992)

Introduction to effect size

How to estimate effect size?

- prior research
- theoretical context of the research

  "What is the smallest, clinically significant difference?"

  "What is important enough to warrant attention?"
- use of special conventions

Power and other factors

What affects power?

- type I error
- type II error
- sample size
- effect size

Power and other factors

Power and type I error

- α level is the chance of a type I error
- as α decreases, power decreases
- we want α to be small
- generally α is 0.01 or 0.05

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Power and other factors

Power and type II error

- β is the chance of a type II error
- as β decreases, power increases
- we want β to be small
- generally, $\beta \leq 0.20$

Power and other factors

Power and sample size

- *n* is the number of sampling units in your study
- as sample size increases, power increases
- the more power you want in your study, the larger the sample size you will require

Power and other factors

Power and effect size

- effect size is “the degree to which the null hypothesis is false” (Cohen, 1977)
- as the effect size increases in magnitude, power increases
- we can denote effect size by *d*

Power and other factors

Check your understanding

Types of power analyses

A priori

- done before the study is conducted
- helps in the design of study

Post hoc

- done after the study is conducted
- helps to understand observed results

Compromise

- done when sample size is restricted

How can we estimate sample size?

Analytical formulae

- some exact or approximate formulae available
- typically difficult to obtain

Published tables

- literature has tables of sample size for specific type I error, power, and effect size combinations

  eg. Cohen (1977)

Software

- some software can do power analysis; important to understand inputs and outputs

  eg. G*Power, UnifyPow (SAS)

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Example: scenario

Suppose that you would like to compare the effect of a newly developed drug (drug A) and a current drug (drug B) on systolic blood pressure (sbp). You would like to know how many patients to include in your study.

What information do we need?

Example: statistical problem

What is the hypothesis?

H_O: the mean sbp among patients on drug A is the same as those patients on drug B

ie. $\mu_A = \mu_B$

H_A: the mean sbp among patients on drug A is different from those patients on drug B

ie. $\mu_A \neq \mu_B$

How will we test this hypothesis?

independent two-sample t-test

.with two-sided alternative

Example: What affects sample size?

type I error

Q: How often would you be comfortable with rejecting H_O when there is no difference?

A: 1 out of 20 times

→ $\alpha = 0.05$

Example: What affects sample size?

power

Q: If there is actually a difference, with what probability would you like to detect this difference?

A: want 80% chance of detecting difference

⇨ $1-\beta = 0.80$

Example: What affects sample size?

effect size

Q: What is the smallest, clinically significant difference that you would like to detect?

A: It is known from past studies that patients on the current drug have an average sbp of 125 mmHg with a standard deviation of 20 mmHg. We would like to detect a 10% difference. We assume the standard deviation of sbp using the new drug will be 20 also.

⇨ $d = (125*0.10)/20 \approx 0.63$

Example: estimating sample size

How can we compute sample size using analytical methods?

For an independent two-sided t-test (with two-sided alternative):

$$n \geq 2*(z_{1-\alpha/2} - z_{\beta})^2/d^2$$
$$\geq 2*(1.96 - (-0.84))^2/(0.63)^2$$
$$\geq 40.2$$

⇨ We need at least 41 subjects per group

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Example: estimating sample size

How can we compute sample size using tables?

- reference: Cohen, 1977
- How to read the table:

  a_1 is type I error for one-sided test

  a_2 is type I error for two-sided test

  d is effect size

  `Power` is power

- table shows sample size per group

Example: estimating sample size

How can we compute sample size using tables?

- From the table, we can see that we need a sample size that is more than 33, but less than 45 in each group
- Using linear interpolation, an effect size of 0.63 corresponds to a sample size of about 41.4

⇨ We need at least 42 subjects per group

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Example: estimating sample size

How can we compute sample size using software?

G*Power (Erdfelder, et al 1996)

- general power analysis program
- can compute sample size and power for t-tests, F-tests, chi-square tests
- freeware for Windows and MacIntosh
- website: http://www.psycho.uni-duesseldorf.de/aap/projects/gpower/

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Example: estimating sample size

How can we compute sample size using software?

1. Select type of test: **`t-Test (means)`** default
2. Select type of analysis: **`A priori`**
3. Select alternative hypothesis: **`Two tailed`**
4. Input parameters:

   effect size = **`0.63`**

   alpha = **`0.05`**

   power = **`0.80`**
5. Hit "enter"

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Example: estimating sample size

What does G*Power output tell us?

- **`Total sample size`** is 82
- **`Actual power`** is 0.8046
- **`Critical`** t(80) is 1.9901

⇨ We need at least 41 subjects per group

Example: follow-up

How do the following affect sample size?

1. want the chance of finding an effect, if one really exists, to be 0.95
2. want to detect a 15% difference
3. standard deviation of sbp ranges from 15-25 mmHg
4. want 99% chance of correctly claiming there is no difference between the two drugs
5. want to see if drug B lowers sbp more than drug A

Example: follow-up part 1

Power, $1-\beta$

$1-\beta = 0.95$, so $\beta = 0.05$

Using analytical methods:

$$n \geq 2*(z_{1-\alpha/2} - z_{\beta})^2/d^2$$
$$\geq 2*(1.96 - (-1.64))^2/(0.63)^2$$
$$\geq 66.5$$

➡ We need at least 67 subjects per group.

Example: follow-up part 2

Effect size, *d*

$d = (125*0.15)/20 \approx 0.94$

Using analytical methods:

$n \geq 2*(z_{1-\alpha/2} - z_{\beta})^2/d^2$

$\geq 2*(1.96 - (-0.84))^2/(0.94)^2$

≥ 17.9

⇨ We need at least 18 subjects per group.

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Example: follow-up part 3

Effect size, *d*

d ranges from 0.50 to 0.83

[$d = (125*0.10)/15 \approx 0.83$; $d = (125*0.10)/25 \approx 0.50$]

Using software:

We can see how sample size changes with effect size

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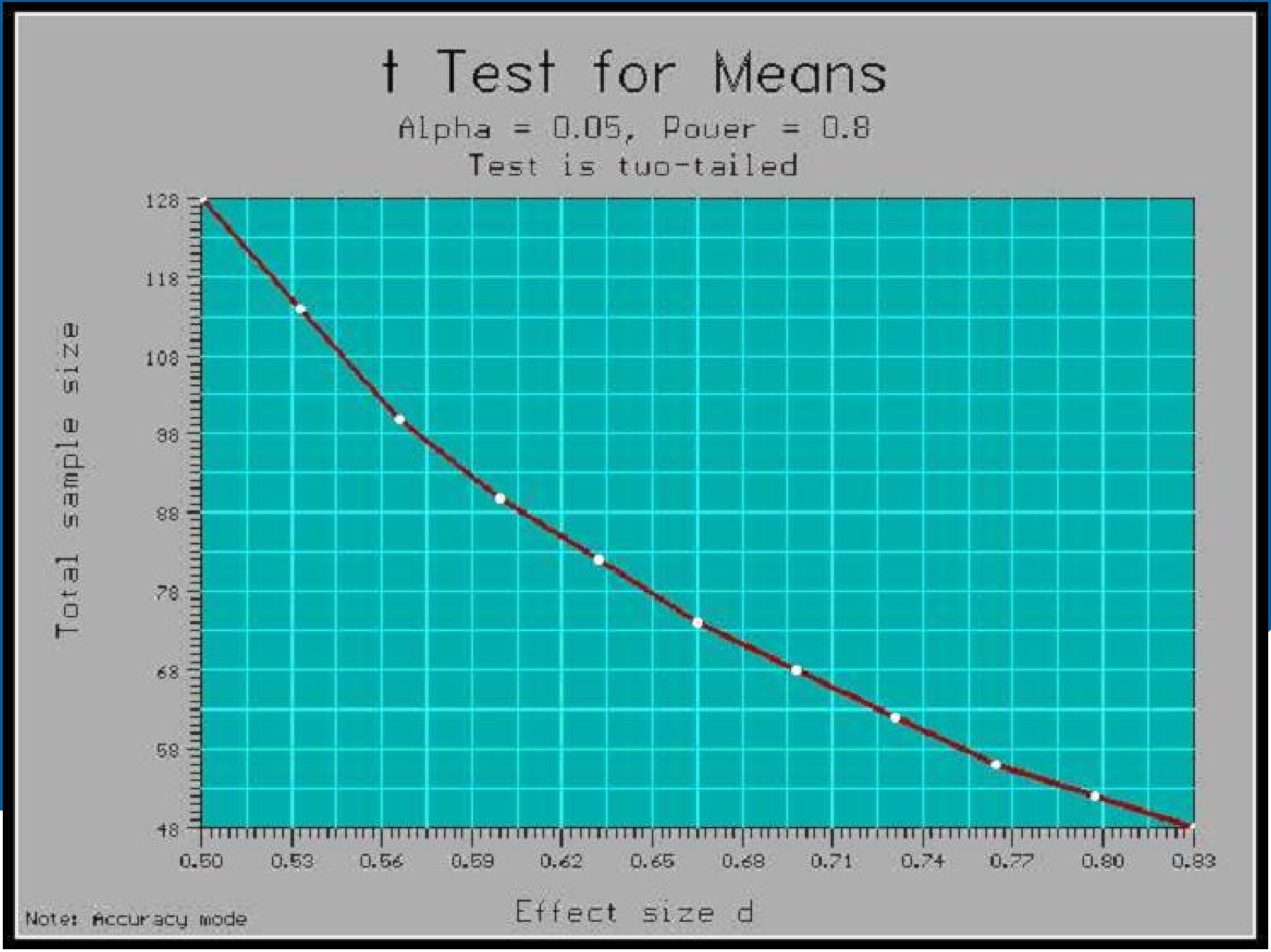
Example: follow-up part 3



Example: follow-up part 4

type I error, α

$\alpha = 0.01$

Using published tables:

$\beta = 0.20$, $d = 0.63$, $\alpha = 0.01$

➡ We need at least 61 subjects per group.

Example: follow-up part 5

hypothesis

one-sided alternative

H_O: $\mu_A \leq \mu_B$

H_A: $\mu_A > \mu_B$

Using software:

Select alternative hypothesis: **`One tailed`**

➡ We need at least 32 subjects per group.

References

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